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THE THEORY OF THE ABSORPTION OF FLAME RADIATION BY MOLECULAR BANDS

by

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THE THEORY OF THE ABSORPTION OF FLAME RADIATION

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ABSTRACT

The absorption of radiation by molecular bands is calculated when the source emits its energy in discrete spectral lines. The results are compared with those for a blackbody source. The absorption of flame radiation by the molecular bands of an absorbing medium is derived for a number of different conditions when the spectral lines do not overlap. These results are then extended to the case when the overlapping of the spectral lines in either the source or absorber can be described either by the Elsasser model or by the statistical model. When the same spectral lines are involved in the emission and absorption processes, the absorptance of flame radiation is always greater than the absorptance of blackbody radiation by the same medium. All of the above results are derived both for the case when the absorbing medium is homogeneous and when the pressure, temperature and amount of absorbing gas vary along the path.

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I. Introduction

The problem of the transmission by the atmosphere of radiation from a continuous blackbody source has been studied extensively in the literature for a wide variety of conditions (e.g. reference 1). When the source is a flame or other substance which emits radiation in discrete spectral lines, the transmittance of this radiation through a gaseous medium may be very different than when a blackbody source is used. Although the transmission of flame radiation by the atmosphere is a problem of considerable practical importance, it has not been discussed to any appreciable extent in the published literature.

In Section II a general theorem is derived which relates the absorptance of flame radiation to the absorptances of the source and absorber media considered separately as well as the absorptance of these two media considered as cells in series. The latter quantity was considered in detail in an article[2] which will be referred to as I. The same notation is used in the present article together with many of the equations from I. This material is not repeated here, so that the reader should first study I.

The absorptance of flame radiation emitted by a series of nonoverlapping spectral lines and absorbed by these same spectral lines in the gaseous medium is studied in Section III. Numerous examples illustrate the differences between the absorptance of flame radiation and that from a blackbody source. Similar results are derived in Section IV when the spectral lines overlap and can be represented either by the Elsasser or the statistical model. The problem of the absorption of flame radiation by the molecules along an atmospheric slant path is discussed in Section V.

II. General Results about the Absorptance of Flame Radiation

The radiation emitted at a particular frequency ν by a source of line radiation (indicated by the subscript ℓ) is given by

$$I = I_b(\nu)\ [1-\exp(-\sum_i k_\ell^{(i)} u_\ell)]\ , \tag{1}$$

where I_b is the spectral irradiance of a blackbody, $k_\ell^{(i)}$ is the absorption coefficient of the i^{th} absorption line, and u_ℓ is the mass of emitting gas per unit area. An expression for $k_\ell^{(i)}$ is given by Eqs. (2) and (3) of I. The integral of Eq. (1) over all frequencies gives the total irradiance of the line source of radiation.

The absorptance of this line radiation by another medium is defined as the ratio of the intensity of the beam which emerges from the absorbing medium to the intensity incident upon the medium. Thus the absorptance is given by

$$A = \frac{\int_{\Delta\nu} [1-\exp(-\sum_i k_\ell^{(i)} u_\ell)]\ [1-\exp(-\sum_i k_a^{(i)} u_a)] d\nu}{\int_{\Delta\nu} [1-\exp(-\sum_i k_\ell^{(i)} u_\ell)] d\nu}\ , \tag{2}$$

where the subscript a refers to the absorbing medium, $\Delta\nu$ is the frequency interval considered, and the factor I_b has been assumed to vary slowly in the interval $\Delta\nu$ and therefore has been removed from the integrals and canceled. The spectral lines in the frequency interval $\Delta\nu$ are numbered in order with the index i. Spectral lines with the same value of i have the same frequency ν_{oi}, for their line enters. The lines in the emitting and absorbing media may have different intensities. If a spectral line occurs in one medium, but not in another, it is entered with zero intensity in the sum for the latter medium.

If radiation from a blackbody source is absorbed by the gaseous molecules of the emitting medium, then its absorptance A_ℓ is

$$A_\ell \Delta\nu = \int_{\Delta\nu} [1-\exp(-\sum_i k_\ell^{(i)} u_\ell)]d\nu. \tag{3}$$

Similarly the absorptance A_a of the absorbing medium for radiation from a blackbody source is

$$A_a \Delta\nu = \int_{\Delta\nu} [1-\exp(-\sum_i k_a^{(i)} u_a)]d\nu. \tag{4}$$

If radiation from a blackbody source is absorbed first by the molecules of the emitting medium and then by those in the absorbing medium, this is identical to the case of the absorption of radiation by two cells in series as studied in detail in I. The absorptance of both media for blackbody radiation is

$$A_{\ell a}\Delta\nu = W_{\ell a} = \int_{\Delta\nu} [1-\exp(-\sum_i k_\ell^{(i)} u_\ell - \sum_i k_a^{(i)} u_a)], \tag{5}$$

where W is the equivalent width of two cells in series as defined by Eqs. (1) and (29) of I.

The absorptance A of radiation from a line source can be written from Eqs. (2) - (5) as

$$A = (A_\ell + A_a - A_{\ell a})/A_\ell . \tag{6}$$

This very general result relates A to the absorptances (A_ℓ and A_a) of the gases composing the line source and the absorbing material each considered separately and their combined absorptance $A_{\ell a}$ for blackbody radiation. Since all of these expressions have been derived previously for many different conditions, it is often a simple matter to calculate the absorptance of line radiation. Equation (6) is valid for any line shape, any frequency interval $\Delta\nu$, and any variation of the line spacing, intensity, or half-width within the band.

It is interesting to rewrite Eq. (6) in terms of the transmittance τ. We find that

$$\tau = (\tau_a - \tau_{\ell a})/(1-\tau_\ell), \tag{7}$$

where

$$\tau_\ell = 1-A_\ell$$

and similarly for the other transmittances. This equation is valid in the limit when a line source becomes a blackbody source. By definition all radiation incident upon a blackbody is absorbed so that $\tau_\ell = \tau_{\ell a} = 0$.

III. Nonoverlapping Spectral Lines

When there is no appreciable overlapping of the spectral lines, results can readily be derived for the four limiting cases which correspond to all possible combinations of describing the source and the absorber respectively by the weak or the strong line approximations. The expressions for the absorptance of a single medium in the weak and strong line limits are well known (e.g. reference 3). The equations for the absorptance when blackbody radiation passes through both media in series are derived in I.

When both the source and absorber can be described by the weak line approximation, it follows from Eq. (6) together with Eq. (10) of I that

$$A = \frac{S_a u_a}{\pi(\alpha_\ell + \alpha_a)} = x_a \frac{2}{1+(\beta_\ell/\beta_a)}, \tag{8}$$

where

$$x = Su/2\pi\alpha,$$

$$\beta = 2\pi\alpha/d,$$

α is the half-width, and d is the mean line spacing (which cancels out of all equations for nonoverlapping lines). For convenience, Eq. (8) and the other equations of this section are written for a single line only, since the absorptance can be calculated separately for each nonoverlapping line and then averaged over the appropriate frequency interval. It is interesting to note that Eq. (8) is independent of the amount of emitting gas in the line source. When the weak line approximation is valid, the same fractional part of the radiation is absorbed by the second medium for any amount of emitting gas.

When both the source and absorber can be described by the strong line approximation, it follows from Eq. (6) together with Eq. (15) of I that

$$A = 1 + \frac{\beta_a x_a^{\frac{1}{2}}}{\beta_\ell x_\ell^{\frac{1}{2}}} - \left(1 + \frac{\beta_a{}^2 x_a}{\beta_\ell{}^2 x_\ell}\right)^{\frac{1}{2}} . \tag{9}$$

When the source can be described by the weak line approximation and the absorber by the strong line approximation, it follows similarly from Eq. (19) of I that

$$A = 1 - e^{\eta}\ \left[1-\phi(\eta^{\frac{1}{2}})\right], \tag{10}$$

where

$$\phi(x) = 2\pi^{-\frac{1}{2}}\int_0^x e^{-y^2}\,dy$$

and

$$\eta = 2\beta_\ell{}^{-2}\beta_a{}^2 x_a = S_a\alpha_a u_a/\pi\alpha_\ell{}^2 .$$

The following limiting forms are often convenient

$$A = 2\pi^{-\frac{1}{2}}\,\eta^{\frac{1}{2}}, \qquad \eta << 1, \tag{11}$$

and

$$A = 1 - \pi^{-\frac{1}{2}}\eta^{-\frac{1}{2}}, \qquad \eta \gg 1. \tag{12}$$

When the source can be described by the strong line approximation and the absorber by the weak line approximation, it follows from Eq. (25) of I that

$$A = \pi^{\frac{1}{2}} \xi^{-\frac{1}{2}} x_a \left\{1-e^{\xi}[1-\phi(\xi^{\frac{1}{2}})]\right\}, \tag{13}$$

where

$$\xi = 2\beta_a^{-2}\beta_\ell^2 x_\ell$$

The limiting forms are

$$A = 2x_a, \qquad \xi \ll 1, \tag{14}$$

and

$$A = \pi^{\frac{1}{2}}\xi^{-\frac{1}{2}} x_a(1-\pi^{-\frac{1}{2}}\xi^{-\frac{1}{2}}), \qquad \xi \gg 1. \tag{15}$$

The following examples illustrate the variation of the absorptance under various conditions. The variation of the absorptance as a function of x_a is shown in Fig. 1 when the line source can be described by the weak line approximation (x_ℓ may have any value less than 0.2). Curves are given for various values of $q = \beta_a/\beta_\ell$. For a given value of x_a, the absorptance approaches a limiting value as q becomes large. This is known as the weak line limit and occurs when the pressure in the region where the absorber molecules occur is much larger than the pressure where the line emitter molecules occur ($\beta_a \gg \beta_\ell$). For any particular value of q the absorptance at first increases linearly with x_a. If $q < 1$ there is also a region where the absorptance increases as $x_a^{\frac{1}{2}}$. If $q > 1$, the absorptance is nearly complete whenever $x_a > 2$, i.e. when the absorbing medium can be described by the strong line approximation.

The absorptance as a function of $q^2 x_a$ is shown in Fig. 2 when the line source can be described by the weak line approximation (x_ℓ may have any value less than 0.2). The uppermost limiting curve on this plot is the strong line limit. When $q < 1$, there is a region of the absorptance curve which varies as $x_a^{\frac{1}{2}}$. The absorptance varies as $x_a^{\frac{1}{2}}$ only when $x_\ell < 0.2$ and $x_a > 2$ and at the same time when the spectral lines in the absorbing medium completely absorb all of the incident radiation at their line centers, while the radiation from the line source is sufficiently broadly distributed in frequency to provide significant radiation at all frequencies where the line in the absorbing medium is capable of interacting with the incident radiation, thus requiring $q = \beta_a/\beta_\ell << 1$.

An illustrative example when the line source is represented by the strong line approximation ($x_\ell = 100$) is shown in Fig. 3. The absorptance is plotted as a function of $q^2 x_a$. Again the uppermost limiting curve is the strong line limit. The same remarks which are given for Fig. 2 also apply to this case. The main difference between these two figures lies in the numerical values. It is necessary to have considerably larger values of x_a in the present case in order to have the same absorptance as shown in Fig. 2. This is because the line source at the same pressure is emitting the radiation over a considerably wider frequency interval when $x_\ell = 100$ than when $x_\ell < 0.2$. In order to absorb the radiation efficiently when $x_\ell = 100$ either the pressure in the absorbing medium must be considerably larger than that in the emitter or there must be a large amount of absorbing gas. The absorptance curves in Figs. 2 and 3 are identical when $q = 100$, i.e. the lines of the absorbing molecules are sufficiently broadened to absorb efficiently over the spectrum of emitted frequencies. On the other hand, if $q = 0.1$, the absorptance equals 0.1 in Fig. 2 when $x_a = 0.7$, but it equals 0.1 in Fig. 3 only when $x_a = 120$. The very much larger value of x_a is required in the latter case in order to produce the same absorptance.

How much more efficiently does a line absorber absorb the radiation from a line source compared to a blackbody source? The ratio of the absorptance of a line source to that of a blackbody source is plotted in Fig. 4 as a function of x_a. Two cases are shown: (1) $x_\ell < 0.2$; (2) $x_\ell = 10$. The particular values $q = 1$ and $\beta_\ell = 0.0314$ were chosen. For this calculation to be meaningful it is important to use the correct equations for the absorptance over a finite frequency interval as tabulated in reference 4. Of course it is assumed that the line centers of the emitting and absorbing molecules coincide, although their intensities and half-widths may be different. It is seen from Fig. 4 that the absorptance for a line source is always greater than that for a blackbody source for the same conditions in the absorbing medium. The molecules can absorb most efficiently at just those frequencies where the radiation is preferentially emitted by the source. Thus if the absorption of radiation by a line absorber is compared for a line source and blackbody source under conditions such that they are emitting equal amounts of power in a given frequency interval, a greater amount of radiation is always absorbed from the line source. Figure 4 shows that under some conditions that this may be a significantly greater amount of radiation.

IV. Overlapping Spectral Lines

The general results of Section II represented by Eqs. (6) and (7) are equally applicable when the spectral lines overlap. In this section we substitute previously derived expressions for band absorptance into these equations in order to study the absorptance of radiation from a line emitter by overlapping spectral lines.

A. Elsasser model

When the strong line approximation is valid, the absorptance in a single absorbing medium of radiation from a blackbody source for an Elsasser band is

$$A_a = \phi[(\tfrac{1}{2}\beta_a{}^2 x_a)^{\frac{1}{2}}], \tag{16}$$

with a similar expression for A_ℓ. The function ϕ is defined after Eq. (10). When these expressions are substituted into Eq. (6) together with the value of $A_{\ell a}$ obtained from Eq. (47) of I, we find that

$$A = \left\{\phi(\chi_\ell) + \phi(\chi_a) - \phi[(\chi_\ell{}^2 + \chi_a{}^2)^{\frac{1}{2}}]\right\} / \phi(\chi_\ell), \tag{17}$$

where

$$\chi = (\tfrac{1}{2}\beta^2 x)^{\frac{1}{2}} . \tag{18}$$

The variation of the absorptance for an Elsasser band as a function of $\chi_a = (\frac{1}{2}\beta_a{}^2 x_a)^{\frac{1}{2}}$ is shown in Fig. 5. For a given value of χ_a the absorptance is always greater for a line source than for a blackbody source. When $\chi_\ell > 1$, the absorptance for both sources is nearly the same. In this case the overlapping of the spectral line is sufficient to make the line source nearly identical to a blackbody source. However, when $\chi_\ell < 1$, there may be large differences in the absorptance of radiation from a line source compared to a blackbody source. The curves in this figure are only valid when both the source and absorber media can be described by the strong line approximation.

B. Statistical Model

The derivation of an expression for the transmittance of radiation from a line source when the statistical model is valid follows the same steps as the derivation for a blackbody source (e.g. reference 5). For this model the expressions for the transmittance are somewhat less cumbersome to write than those for the absorptance. For a line source it follows from Eq. (2) and from Eq. (17) of reference 5 that the transmittance is

$$\tau = \frac{\int_{\Delta\nu}\cdots\int_{\Delta\nu} d\nu^{(1)}\cdots d\nu^{(N)}\int_0^\infty\cdots\int_0^\infty P(S^{(1)})dS^{(1)}\cdots P(S^{(N)})dS^{(N)}\left[1-\exp\left(-\sum_{i=1}^{N} k_\ell^{(i)}u_\ell^{(i)}\right)\right]\exp\left[-\sum_{i=1}^{N} k_a^{(i)}u_a^{(i)}\right]}{\int_{\Delta\nu}\cdots\int_{\Delta\nu} d\nu^{(1)}\cdots d\nu^{(N)}\int_0^\infty\cdots\int_0^\infty P(S^{(1)})dS^{(1)}\cdots P(S^{(N)})dS^{(N)}\left[1-\exp\left(-\sum_{i=1}^{N} k^{(i)}u^{(i)}\right)\right]}, \tag{19}$$

where N spectral lines occur in the frequency interval $\Delta\nu$. The quantity $P(S^{(i)})$ expresses the probability distribution function of the ith spectral line. When this expression for the transmittance is evaluated as in the usual derivation for the statistical model, it is found that[5]

$$\tau = \frac{\bar{\tau}_a^{\,N} - \bar{\tau}_{\ell a}^{\,N}}{1 - \bar{\tau}_\ell^{\,N}}, \tag{20}$$

where

$$\begin{aligned}\bar{\tau} &= \int_0^\infty P(S)\tau dS \\ &= \int_{\Delta\nu} d\nu \int_0^\infty P(S)dS e^{-ku}.\end{aligned} \tag{21}$$

The quantity $\bar{\tau}$ is the transmittance of an isolated spectral line over the frequency interval $\Delta\nu$ and is averaged over the distribution of line intensities within this same frequency interval.

When the number of spectral lines N is large, the quantities which appear in Eq. (20) can be approximated by an exponential[5] so that

$$\tau = \frac{\exp(-\bar{W}_a/d) - \exp(-\bar{W}_{\ell a}/d)}{1 - \exp(-\bar{W}_\ell/d)}, \tag{22}$$

where $\bar{W}_a$, $\bar{W}_\ell$, and $\bar{W}_{\ell a}$ are the equivalent widths averaged over the intensity distribution for the absorbing medium, the emitting medium, and the two media in series respectively.

When the statistical model is valid, the absorptance for a line source can readily be found from either Eq. (20) or Eq. (22) and the appropriate expression for the equivalent width of a single spectral line, W. For a single medium (e.g. reference 5)

$$A = 1-\tau = W/d = \beta f(x), \tag{23}$$

where

$$f(x) = xe^{-x}[I_o(x)+I_1(x)], \tag{24}$$

I_o and I_1 are the Bessel functions of imaginary argument. For radiation which passes in series through the source medium and the absorbing medium. the equivalent width $W_{\ell a}$ for a single spectral line can be obtained from Eqs. (7), (10), (15), (17), (19), and (25) of I.

For example, when both media can be represented by the weak line approximation, the transmittance can be obtained from either Eq. (20) or (22) with

$$W_a/d = 1-\tau_a = \beta_a x_a - \tfrac{1}{2}\beta_a x_a^2 \tag{25}$$

together with a similar expression for W_ℓ and with $W_{\ell a}$ obtained from Eq. (10) of I. In this latter equation the symbol for the equivalent width of a single line includes terms through x^2.

On the other hand when both media can be represented by the strong line approximation, $W_{\ell a}$ is obtained from Eq. (15) of I.

V. Slant Path Absorption

In the previous sections it has been assumed that the absorbing medium is homogeneous. When the pressure, temperature, and amount of absorbing gas vary along the path, the treatment becomes more complicated. However, the same method as is used in the preceding sections can be combined with the usual treatment for the absorption of blackbody radiation along a slant path.[6]

The absorptance of radiation from a line source by absorbing molecules along a slant path is defined by

$$A = \frac{\int_{\Delta\nu}[1-\exp(-\sum_i k_\ell^{(i)} u_\ell)][1-\exp(-\sum_i \int_0^{u_a} k_a^{(i)} du)]d\nu}{\int_{\Delta\nu}[1-\exp(-\sum_i k_\ell^{(i)} u_\ell)]d\nu} \quad , \tag{26}$$

where the integral over u is along the slant path. If we define A_ℓ by Eq. (3) and let

$$A_a' \Delta\nu = \int_{\Delta\nu}[1-\exp(-\sum_i \int_0^{u_a} k_a^{(i)} du)]d\nu \tag{27}$$

and

$$A_{\ell a}' \Delta\nu = \int_{\Delta\nu} [1-\exp(-\sum_i k_\ell^{(i)} u_\ell - \sum_i \int_0^{u_a} k_a^{(i)} du)]d\nu, \tag{28}$$

where the prime indicates that the quantities are evaluated along a slant path. From Eqs. (3), (26), (27), and (28) it follows that

$$A = (A_\ell + A_a' - A_{\ell a}')/A_\ell, \tag{29}$$

which has the same form as Eq. (6). Since many expressions for the absorptance A_a' along a slant path are known[6] which are valid under many different conditions, the absorptance for a line source can be obtained immediately from Eq. (29, whenever $A_{\ell a}'$ is also known.

Actually all of the derivations of $A_{\ell a}$ which are given in Sections II, III, and IV of reference I can be repeated when $A_{\ell a}'$ is given by Eq. (28) instead of Eq. (5). These generalizations are so straightforward that only a few results are summarized here.

When the weak line approximation is valid for both the emitting and absorbing media, an expansion of the exponential shows that Eq. (7) of I is still valid.

When terms through the second order are retained, Eq. (10) of I is replaced by

$$A'_{\ell a} = A_\ell + A'_a - \frac{S_\ell u_\ell}{\pi \Delta\nu} \int_0^{u_a} \frac{S_a(u)}{\alpha + \alpha_a(u)} \, du, \tag{30}$$

where S_a and α_a are considered as functions of u as they vary along the slant path. Thus from Eqs. (3), (29), and (30) we find for the absorptance

$$A = \frac{1}{\pi} \int_0^{u_a} \frac{S_a(u)}{\alpha_\ell + \alpha_a(u)} \, du \; . \tag{31}$$

When both regions can be represented by the strong line approximation, the same derivation as lead to Eq. (15) of I shows that

$$A'_{\ell a} = (A_\ell^{\,2} + A_a'^{\,2}) \, , \tag{32}$$

which can be written explicitly as

$$A'_{\ell a} \Delta\nu = 2(S_\ell \alpha_\ell u_\ell + \int_0^{u_a} S_a \alpha_a du)^{\frac{1}{2}}, \tag{33}$$

by the use of the usual strong line expressions.[3,6]

When the emitting and absorbing media can be represented by the weak line and strong line approximations respectively, the same derivations which lead to Eq. (19) of I shows that

$$A'_{\ell a} = A'_a + A_\ell e^{\eta} [1 - \phi(\eta^{\frac{1}{2}})], \tag{34}$$

where

$$\eta = \pi^{-1} \alpha_\ell^{\,-2} \int_0^{u_a} S_a u_a du, \tag{35}$$

and ϕ is defined after Eq. (10). Thus the absorptance from Eq. (29) is

$$A = 1-e^{n}[1-\phi(n^{\frac{1}{2}})]. \tag{36}$$

When the emitting medium can be represented by the strong line approximation and the absorbing medium by the weak line approximation, Eq. (25) of I is still valid if the appropriate expression for slant path absorptance is used for the equivalent width in the second medium.

The overlapping of the spectral lines can easily be taken into account when the statistical model is valid. The expressions derived in this section for the absorptance of a single spectral line are merely substituted into either Eq. (20) or Eq. (22).

The equations for the Elsasser model can also be generalized for absorption along an atmospheric slant path. For example, when the strong line approximation is valid, the same derivation as lead to Eq. (46) of I shows that

$$A'_{\ell a} = \phi[(\pi/d)^{\frac{1}{2}} \; (S_\ell \alpha_\ell u_\ell + \int_0^{u_a} S_a \alpha_a du)^{\frac{1}{2}} \;]. \tag{37}$$

All of the equations in this section can also be generalized to include the case when the emitting medium is not homogeneous. Since the derivations are exactly the same as for the equations already discussed, these results are not explicitly presented here.

VI. Conclusions

The absorption of radiation from a source composed of a discrete number of spectral lines has been calculated for a number of different conditions. First of all it was shown that the absorptance of flame radiation can be related by Eq. (6) to the individual absorptance values for blackbody radiation of the emitting and absorbing media together with their absorptance when considered as two cells in series.

When the overlapping of the spectral lines can be neglected, the absorptance of flame radiation is given by Eqs. (8), (9), (10), and (13). When the overlapping can be represented by the Elsasser model, the absorptance is given by Eq. (17). The absorptance for the statistical model can be found from Eq. (20) or (22).

When the pressure, temperature, and concentration of the absorbing gas vary along the path, the absorptance of flame radiation can be obtained from Eqs. (29), (31), (33), (34), (36), and (37).

When the same spectral lines are involved in the emission and absorption processes, it is shown that the absorptance of flame radiation is always greater than the absorptance of blackbody radiation by the same mecium. The radiation is absorbed most efficiently near the line centers of the absorbing molecules which coincide with the frequencies at which the radiation is preferentially emitted by the line source.

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Captions for Figures

Fig. 1. The absorptance of radiation from a line source by nonoverlapping spectral lines as a function of $x_a = S_a u_a / 2\pi\alpha_a$, where the subscript a refers to the absorbing medium. These curves are valid when the line source by itself can be represented by the weak line approximation ($x_\ell < 0.2$). Curves for various values of $q = \beta_a/\beta_\ell$ are shown. The uppermost limiting curve is the weak line limit.

Fig. 2. The absorptance of radiation from a line source by nonoverlapping spectral lines as a function of $q^2 x_a$. These curves are valid when the line source by itself can be represented by the weak line approximation ($x_\ell < 0.2$). The uppermost limiting curve is the strong line limit.

Fig. 3. The absorptance of radiation from a line source by nonoverlapping spectral lines as a function of $q^2 x_a$. Since it is assumed that $x_\ell = 100$, the line source by itself can be represented by the strong line approximation. The uppermost limiting curve is the strong line limit.

Fig. 4. The ratio of the absorptance for a line source to the absorptance for a blackbody source as a function of x_a. It is assumed that $q = 1$ and $\beta_\ell = \beta_a = 0.0314$. For one curve, $x_\ell < 0.2$, and, for the other curve, $x_\ell = 10$. It is assumed that the spectral lines do not overlap.

Fig. 5. The absorptance of radiation from a line source by a band of spectral lines which obeys the Elsasser model as a function of $\chi_a = (\frac{1}{2}\beta_a{}^2 x_a)^{\frac{1}{2}}$. For a given value of χ_a the absorptance is always greater than that from a blackbody source. It is assumed in this figure that both media can be represented by the strong line approximation. Thus the following conditions must both be satisfied: $x_\ell > 2$; $x_a > 2$.

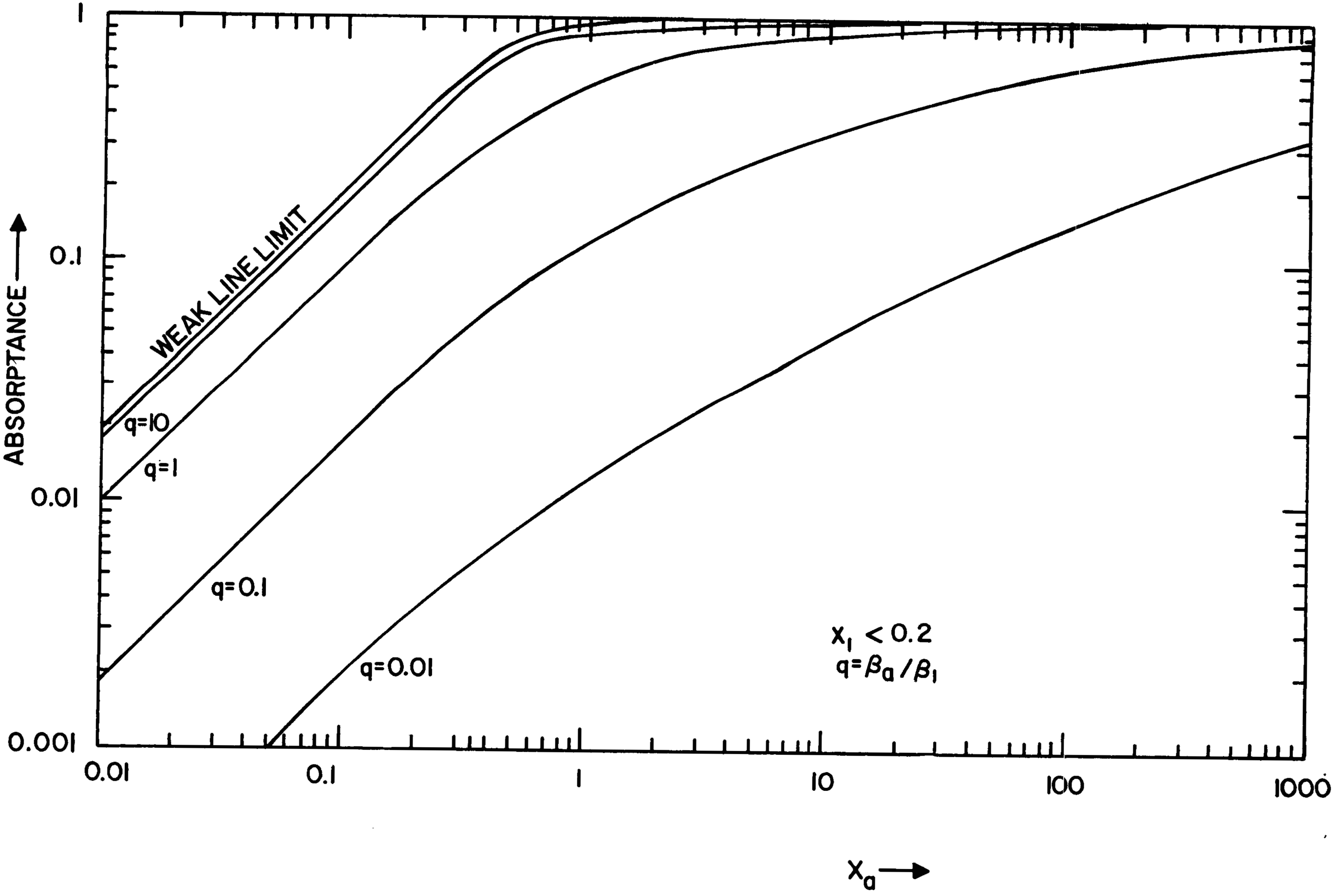
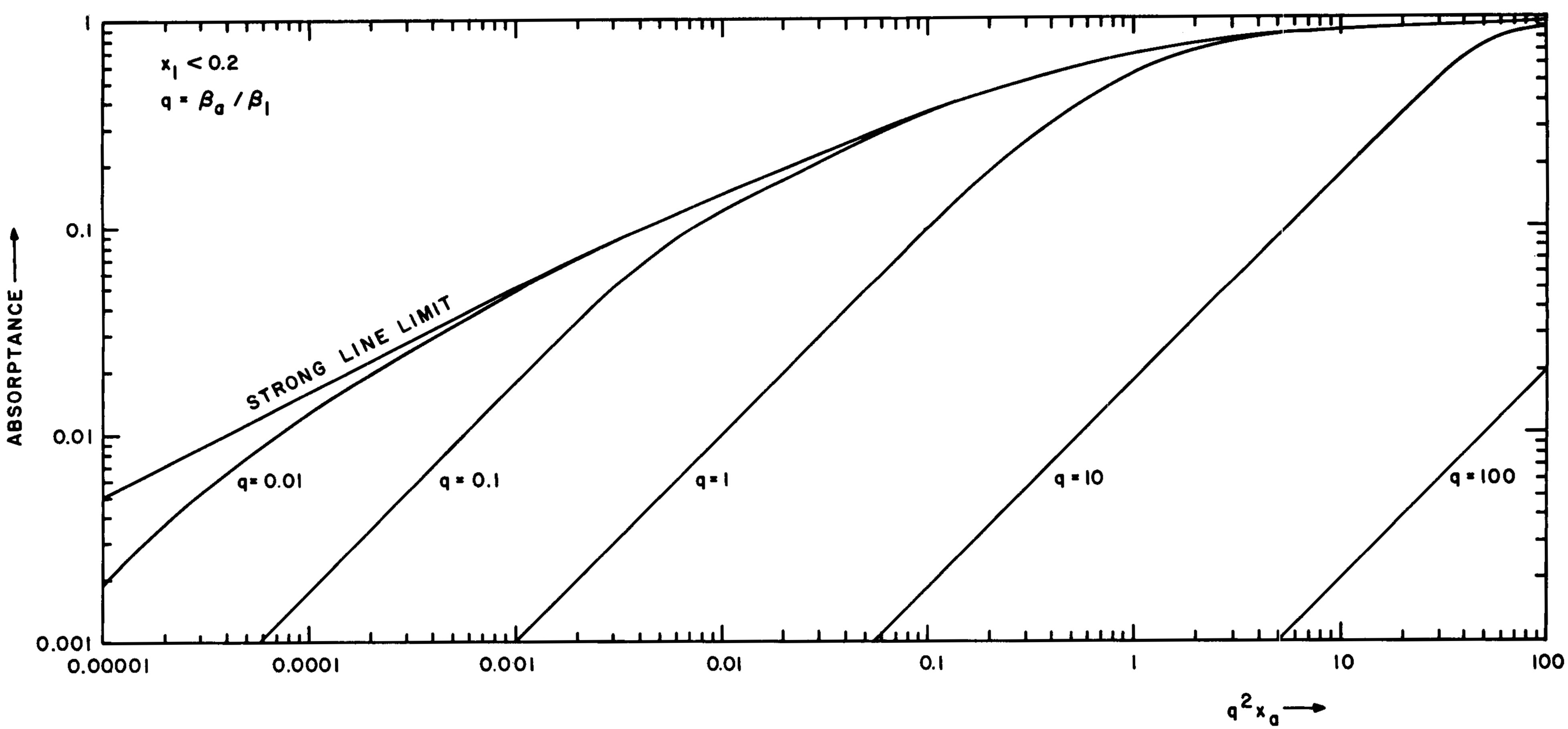
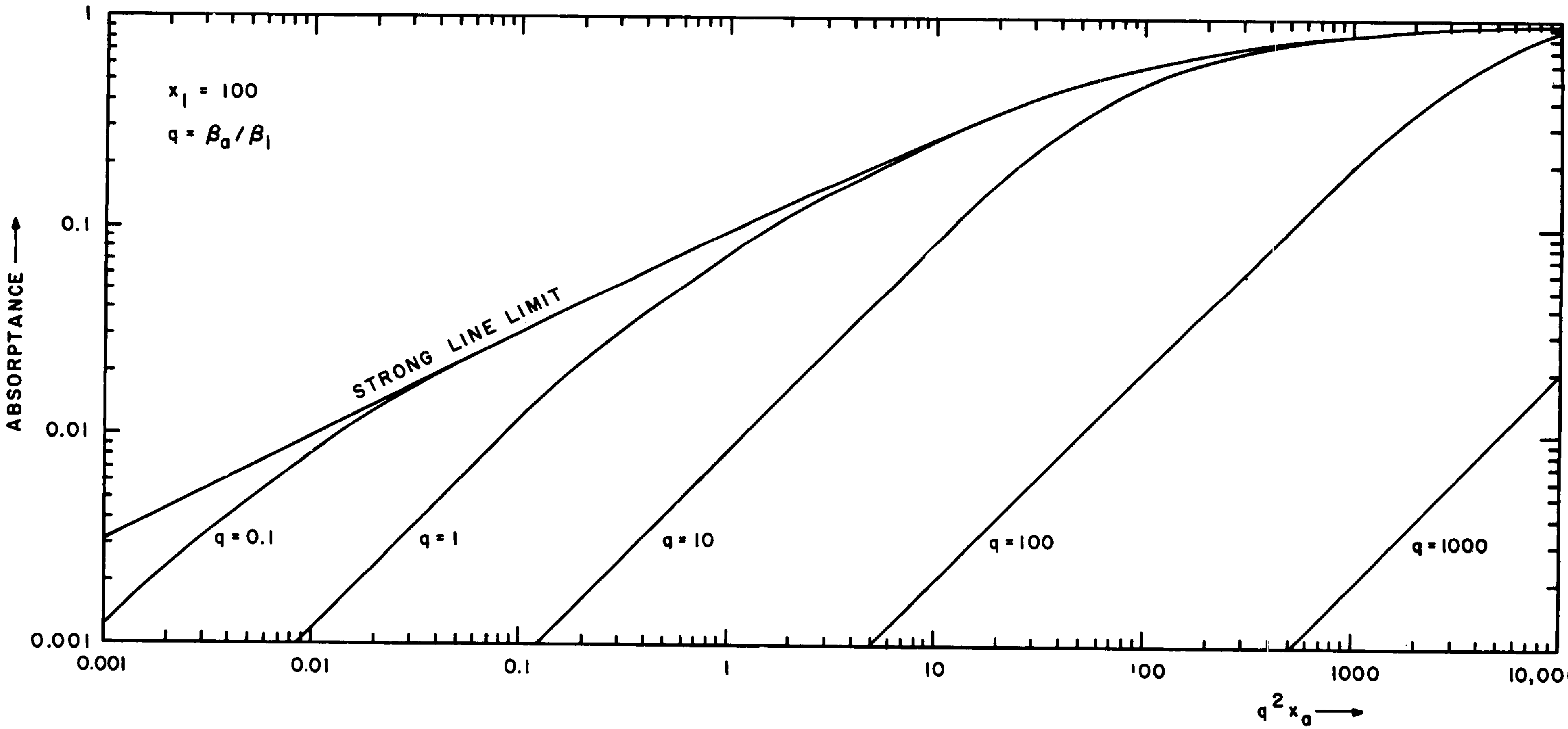

ABSORPTANCE
1
0.1
0.01
0.001
0.01
0.1
1
10
100
1000
WEAK LINE LIMIT
q=10
q=1
q=0.1
q=0.01
$X_1 < 0.2$
$q = \beta_a / \beta_1$
X_a

FIGURE 1



$x_1 < 0.2$
$q = \beta_a / \beta_1$
STRONG LINE LIMIT
q = 0.01
q = 0.1
q = 1
q = 10
q = 100
ABSORPTANCE
1
0.1
0.01
0.001
0.00001
0.0001
0.001
0.01
0.1
1
10
100
$q^2 x_a$

FIGURE 2



$x_1 = 100$
$q = \beta_a / \beta_1$
STRONG LINE LIMIT
q = 0.1
q = 1
q = 10
q = 100
q = 1000
ABSORPTANCE
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10
100
1000
10,000
$q^2 x_a$

FIGURE 3

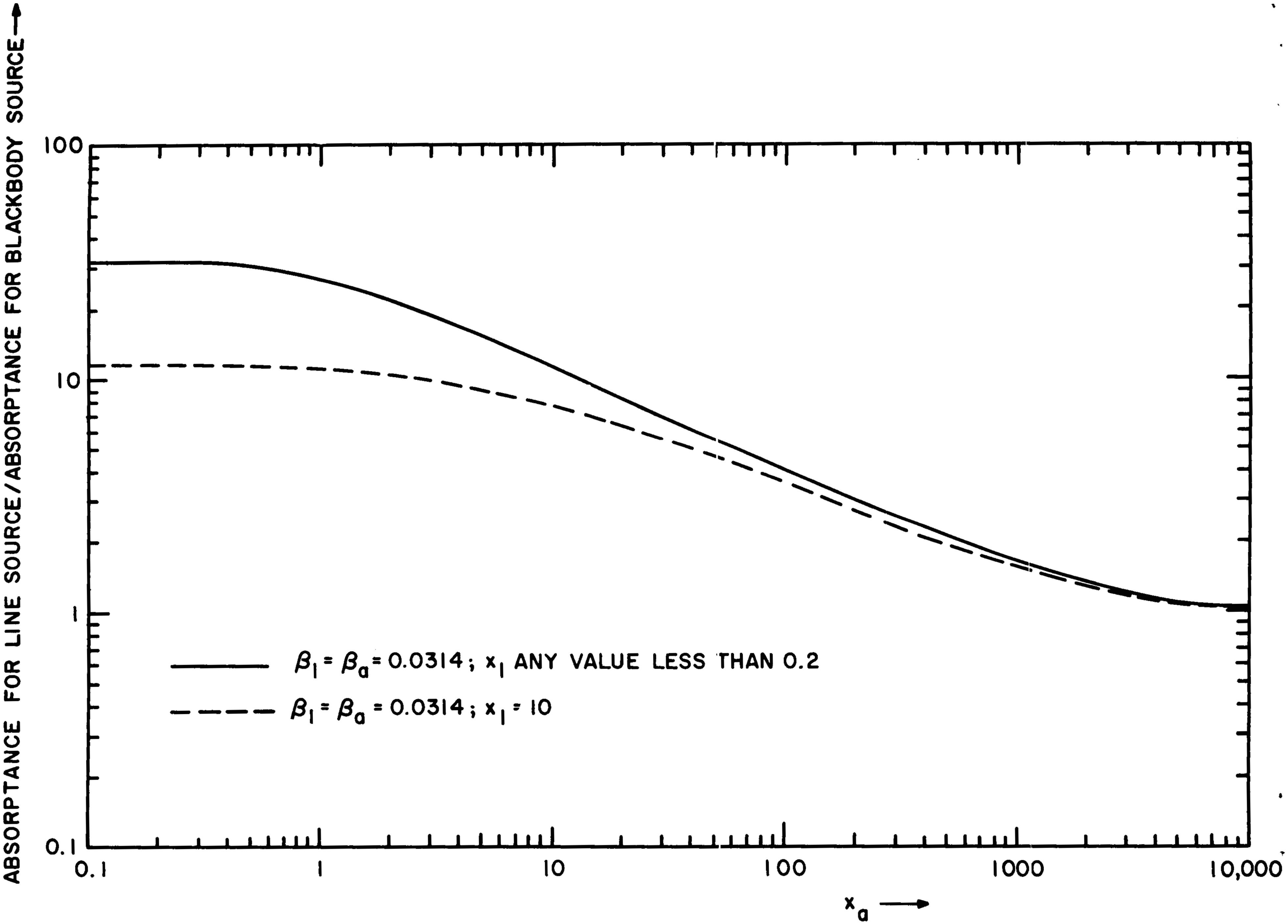

ABSORPTANCE FOR LINE SOURCE/ABSORPTANCE FOR BLACKBODY SOURCE
100
10
1
0.1
0.1
1
10
100
1000
10,000
x_a
$\beta_1 = \beta_a = 0.0314$; x_1 ANY VALUE LESS THAN 0.2
$\beta_1 = \beta_a = 0.0314$; $x_1 = 10$

FIGURE 4

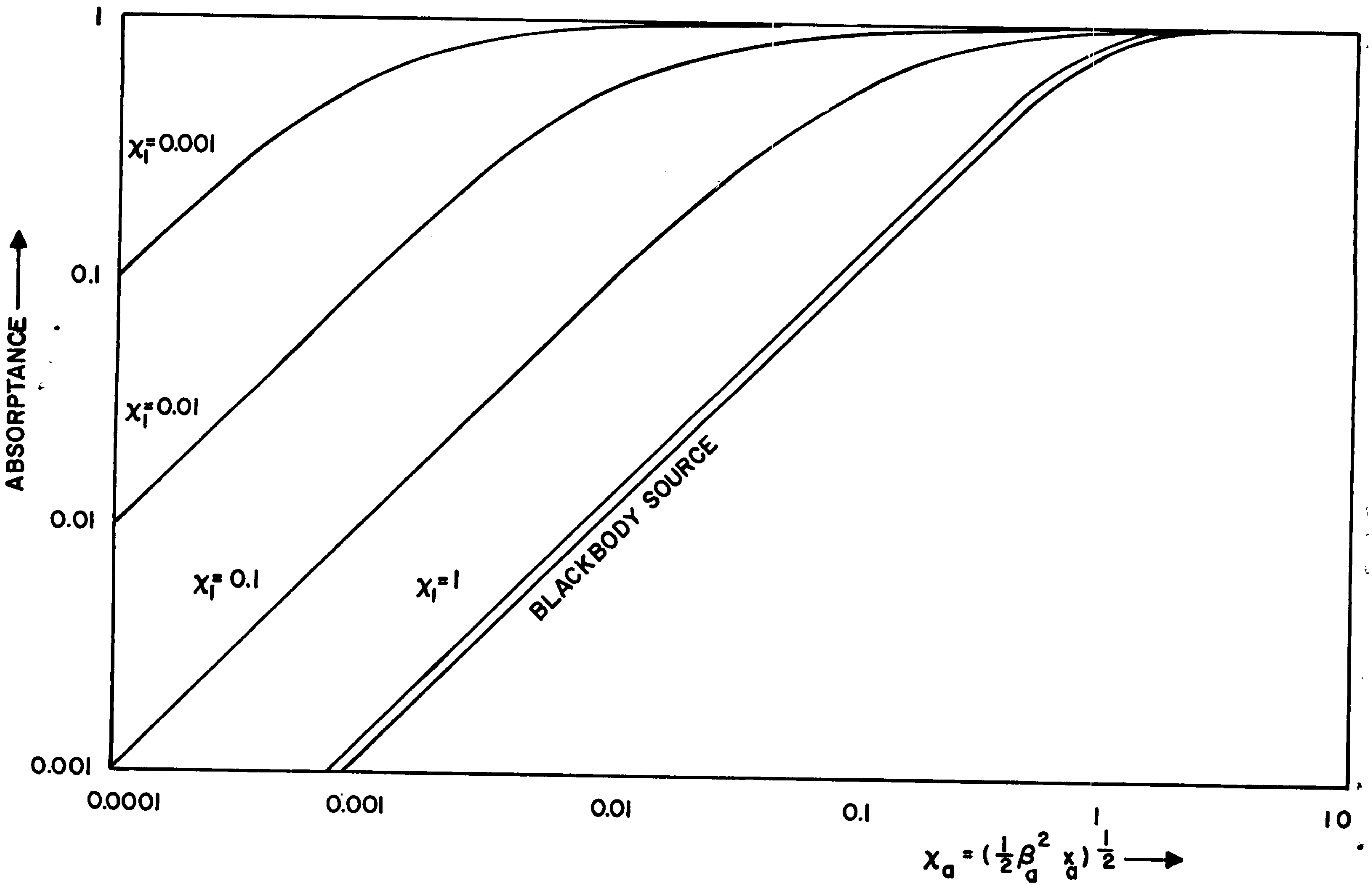

ABSORPTANCE
1
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1
10
$\chi_1 = 0.001$
$\chi_1 = 0.01$
$\chi_1 = 0.1$
$\chi_1 = 1$
BLACKBODY SOURCE
$\chi_a = (\frac{1}{2}\beta_a^2 x_a)^{\frac{1}{2}}$

FIGURE 5